Technical Comment

Comment on "Dynamics of a Cantilever Beam Attached to a Moving Base"

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Introduction

IN light of the strong claims made in Ref. 1, this comment attempts to clarify certain issues and to provide an additional perspective. The detailed model presented in Ref. 1 should prove useful in studying the dynamics of beam-like structures. The authors, however, create the impression that a new theory has been discovered and go to great lengths to point out that existing multibody codes (e.g., DISCOS, ALLFLEX, TREETOPS, NB0D2) do not account for axial displacements that accompany the transverse flexure of a beam. While not identified as such in Ref. 1, this effect is often referred to as foreshortening[2] (Fig. 1) and has, in fact, already been modeled extensively in much of the literature (see Table 1)[2-16] even if it has not been included in all multibodied formulations.

Role, Status of Multibody Codes

The main accomplishment of multibody formulations, to date, has been to model the dynamics of systems of interconnected rigid bodies, arbitrary in number and in shape, having six degrees of freedom at the hinge points and allowing linearly elastic motions. However, no tractable methods of analysis exist for the nonlinear elastodynamics of a generic single, or multibodied, configuration. Consequently, a user familiar with the role and evolution of multibody dynamics modeling would not normally expect to find an effect as specialized as the beam foreshortening included a priori. Interestingly, neither is it included in the work of Kane et. al.[17] dealing with only a spinning beam.

Present day multibody software can, of course, be revised to accommodate foreshortening, but what is needed to justify the additional development effort and computational cost is an answer to the question "when is foreshortening significant and when is it not?" This concern, as it relates to the modeling of orbiting configurations, is also raised earlier in Ref. 18. A partial answer is given in Ref. 19 where it is pointed out that the impact of this effect can be expected to be quite small for the majority of (but not all) applications in space. In fact, it is noted that foreshortening has not played a role in any applications of the TREETOPS code.

Treatment of Kinematic Nonlinearity (Foreshortening)

Although foreshortening is a very real displacement, it can be difficult to represent since coordinates chosen when an elastic body is undeformed become altered by the deformation itself. Rather than work with deformed coordinates, finite strain-displacement relations, and tensor analysis, most authors resort to simplifying assumptions based on a constraint between the undeformed and deformed coordinates to arrive at an approximate estimate for the foreshortening. Ultimately, all methods are based on an application of differential line element theory, which provides the basis, as well, for the constraint expressed in Eq. (19) of Ref. 1. None of this is pointed out in Ref. 1 and, in fact, no guidance at all is given regarding the origin of this main result.

In the absence of any detailed development or discussion of the foreshortening issue in Ref. 1, there remains some question as to why the authors choose to represent the three elastic degrees of freedom (two transverse, one axial) by five variables (u_1, u_2, u_3, s, ζ). Whatever the mathematical niceties, this representation is unnecessary since the foreshortening is not an independent degree of freedom. Also, the use of ζ creates problems computationally because it is time dependent and normally should be updated continuously when evaluating the modal integrals in Eq. (41) of Ref. 1. Integration by parts will not eliminate this problem. An approximate form of quadrature may be used, but there is no evidence to indicate that this is done in Ref. 1.

Reference 1 recognizes the foreshortening as it affects the inertia force, yet does not appear to consider it during evaluation of the elastic strain energies. In light of the work done in Ref. 20, this issue should be addressed if the method of Ref. 1, itself, is to avoid being considered ad hoc. Furthermore, the kinematic nonlinearities introduced by foreshortening are simplified in Ref. 1 to yield equations that are linear in terms of vibration coordinates (but still retain higher order spin effects) as is already the case with most other works.

Treatment of the Literature

Kane et al.[1] attempt to lump together much of the work on beam dynamics under the umbrella of the 1974 review paper of Ref. 21. This effort is unsuccessful since Ref. 21 simply does not contain over "200 references" dedicated to the study of "rigid" bodies with flexible appendages, as cited in Ref. 1. There are questions, as well, regarding the manner in which other works are referred to. For example, Kane et al.[1] state that Refs. 2 and 20 are "in connection with aircraft dynamics." Ref. 20, per se, does not deal with any specific configuration, but rather deals only in a general way with the effect of geometric nonlinearity on beam curvature. Ref. 2, on the other hand, is concerned with rotating beams, but contains material too important and relevant to be referred to solely in this way. In fact, it is this 1977 paper that refers in depth to the foreshortening issue.

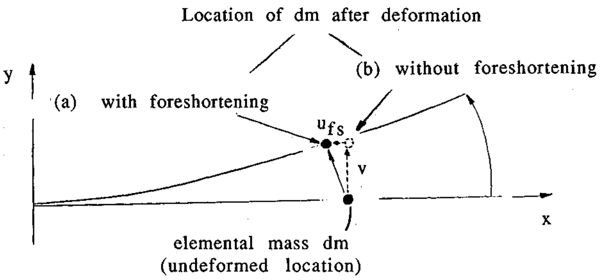

Fig. 1 Concept of axial foreshortening: elemental mass *dm* of a one-dimensional (*x* direction) elastic continuum is constrained to travel along the axial *x* direction (u_{fs}) whenever *dm* experiences a deformation *v* in the transverse *y* direction.

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Table 1 Partial history—Modeling of beam foreshortening effects

	Approach	Dynamic environment	Vibrational equations
Hurty and Rubinstein[3]	Added working displacement (modified beam potential energy)	Centrifugal load (axial direction)	Linear
Meirovitch[4]	Added working displacement (added work function)	General axial load	Linear
Rivello[5]	Added working displacement (modified beam potential energy)	Constant axial load	Linear
Vigneron[6]	Implicit via spatial integrals (kinetic energy)	Three-axis spinning spacecraft	Linear
Budynas[7]	Explicitly in displacement field (kinetic and potential energies)	Three-axis spinning spacecraft, gravity	Linear
Hughes and Fung[8]	Implicit via spatial integration (kinetic energy)	Three-axis spinning spacecraft	Linear
Vigneron[9]	Explicitly in displacement field (kinetic energy)	Three-axis spin	Linear
Lips[10] Lips and Modi[11-16]	Explicitly in displacement field (kinetic and potential energies)	Three-axis spinning spacecraft with mass deployment, gravity	Linear, Second-degree
Kaza and Kvaternik[2]	Explicitly in displacement field (kinetic energy)	Three-axis spin	Second-degree
Kane et al.[1]	Explicitly in displacement field (inertia force via Kane methodology)	Three axis spin, translation	Linear

Note: All methods rely on differential line element theory to relate deformed and undeformed displacements of a one-dimensional beam structure.

Kane et al.[1] allude to seven papers concerned with "the effect of vehicle elasticity on attitude motions." Reference 22 by Likins et al. is included even though it looks only at the dynamics of the spinning beam and of a spinning spring-mass system. More disturbing, the work of Vigneron[9] is a comment on Ref. 22 and does not deal with flex/attitude interactions, but does present an elegant approach for introducing foreshortening and includes the linear vibrational equations inplane and out-of-plane (none of which is pointed out in Ref. 1).

The textbook development given by Meirovitch[4] is not as ad hoc as one is led to believe.[1] It introduces foreshortening in a unified manner through the work function (arising from axial loading) and the techniques of analytical mechanics. This general result is specialized to the case of a beam undergoing constant spin about one axis where the loading of interest is centrifugal and not Coriolis.

The reference to other works as ad hoc by the authors of Ref. 1 becomes even more confusing for the case of Hughes.[23] The use of a Rayleigh-Ritz type of expansion is more an issue related to the solution of the equations of motion and is not a tool for introducing such time varying dynamic effects. This is not the purpose of this approximation in any of Refs. 23–25, nor is it the purpose of Ref. 1 itself (Eqs. 25–27). More surprising at this point is the omission from Ref. 1 of any reference to Hughes and Fung, [8] which applies differential line element theory during the integration process for beam-like components attached to three-axis spinning spacecraft.

The work of Lips[10] and Lips and Modi[11-16] embodies general dynamic models for orbiting deployable beams undergoing three-axis spin. In spite of its relevance, none of this material is referred to by the authors of Ref. 1. These works, which have been widely reported, account for the foreshortening-induced effects for not only the beam dynamic models, but also incorporate it fully into the attitude dynamics for a multibodied cluster configuration (currently contained in computer code GENDYN).

A specific system for which foreshortening is potentially significant is described in Ref. 26. It involves two highly flexible antennas 300 m in length (tip-to-tip) mounted onboard an orbiting, maneuvering Shuttle Orbiter. These results, which include foreshortening in the models, are not referred to in Ref. 1 either.

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Reply by Authors to K. W. London

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As was stated at the outset in the paper[1] that is the subject of Lips' Comment, the paper was written with two objectives in mind, namely 1) to present an algorithm that can be used to predict the behavior of a cantilever beam when the base to which the beam is attached undergoes general three-dimensional motions, and 2) to draw attention to fundamental limitations of certain publicly available multibody computer programs. Because we regard the second of these as more important than the first, we shall deal mainly with those of Lips' statements that appear to be relevant to this issue.

To determine whether or not it was desirable at the time our paper was published to attempt to dispel the notion then widely held in the aerospace industry that one could use certain publicly available computer codes to simulate correctly the motions of multibody systems containing flexible bodies, it is only necessary to recall that this paper and, in particular, the beam spin-up problem described therein, provided, at least in part, the motivation for convening a workshop[2] at the Jet Propulsion Laboratory, Pasadena, California, to examine the role and status of multibody codes. Now, to prevent confusion from arising in connection with issues that were clarified at the workshop, misconceptions underlying the comments made by Lips under the heading of "Role, Status of Multibody Codes" should be pointed out. In particular, attention needs to be refocused on the fact that the flawed multibody codes lead to dynamic softening at *all* rotational speeds at which dynamic stiffening is to be expected, which makes it impossible to determine a priori whether or not a given simulation performed with such a code will be acceptable. This point is so important that we feel obliged to pursue it briefly in the context of a specific example that will allow us to deal incisively with Lips' comments on this issue.

Figure 1a shows a spacecraft supporting two cantilever beams. In Fig. 1b, the spacecraft is represented by a rigid body B; the beams are replaced with massless rods of length l; each rod carries at its end a particle of mass m, and is attached to B at a distance b from B^* (the mass center of B); and linear torsion springs connect the rods to B, each spring having a spring constant of σ units of moment per unit of rotation.

Considering only motions of B during which B rotates about a fixed axis passing through B^*, this motion being characterized as a function of time t by the angle $\phi(t)$, one finds that the exact, fully nonlinear differential equation governing the angle $\theta(t)$ (which plays the role of a beam deflection function) is

$$\ddot{\theta} + \left(\frac{\sigma}{ml^2}\theta + \frac{b}{l}\dot{\phi}^2\sin\theta\right) + \ddot{\phi}\left(1 + \frac{b}{l}\cos\theta\right) = 0 \tag{1}$$

and linearization of this equation in θ leads to

$$\ddot{\theta} + \left(\frac{\sigma}{ml^2} + \frac{b}{l}\dot{\phi}^2\right)\theta + \ddot{\phi}\left(1 + \frac{b}{l}\right) = 0 \tag{2}$$

which is analogous to the equations of motion presented in Ref. 1. By way of contrast, the equation of motion formulation process underlying "conventional" multibody codes, yields the equation

$$\ddot{\theta} + \left(\frac{\sigma}{ml^2} - \dot{\phi}^2\right)\theta + \ddot{\phi}\left(1 + \frac{b}{l}\right) = 0 \tag{3}$$

which differs from Eq. (2) as regards the coefficient of the $\dot{\phi}^2$ term. In Eq. (2), this term has a *stiffening* effect, whereas in Eq. (3) it produces *softening*. Finally if one drops this term altogether, one arrives at what we shall call the "crudely

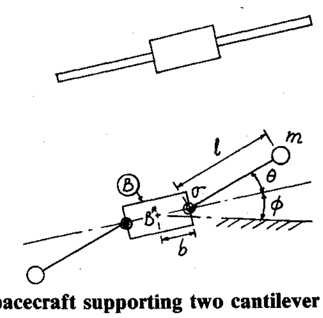

Fig. 1 Spacecraft supporting two cantilever beams.

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